Genetic Algorithms in MATLAB

A Selection of Classic Repeated Games—from Chicken to the Battle of the Sexes

Introduction

In this project, I apply genetic algorithms in MATLAB to several two-player repeated games. The games presented in the analysis are standard to the study of game theory in economics and include the Prisoner's Dilemma, Deadlock, Chicken, the Stag Hunt, the Battle of the Sexes, Matching Pennies, Rock Paper Scissors, Choosing Sides, and a simple pure coordination game. A description of these games and their outcomes is provided in the Economic Model section. The experiments performed in this analysis use the textbook's original setup as a basis for formulating and calibrating several different simultaneous games that are often referenced in introductory microeconomics courses and in the more technical game theoretical literature. I have doubled the textbook's original population size from eight to 16 in order to achieve better convergence to equilibrium in the results. Although I have received only a limited education in the area of game theory, I thought that this pairing of the economic model and the computational method was quite interesting due to its simplicity and its useful contribution to evolutionary game theory. Additionally, the results are consistent with the basic economic theory of simultaneous games involving interactions between two players.

Economic Model

John von Neumann is unequivocally the father of modern game theory. He is best known for his proof of the minimax theorem—that is, an individual can minimize his or her maximum losses by employing a certain pair of strategies in a zero-sum game with perfect information. If each player employs these strategies, the strategies are optimal; John von Neumann showed that the players' minimaxes are equal in absolute value and opposite in sign. Following von Neumann is John Nash. He is best known for his development of non-cooperative game theory and subsequent coining of the Nash equilibrium concept.

A Nash equilibrium is a solution to a non-cooperative game involving two or more players, where each player knows the equilibrium strategies of their opponent, but does not receive any gains from changing their own strategy unilaterally. The corresponding payoffs with these particular strategies result in a pure strategy Nash equilibrium. At the equilibrium, each player employs a strategy that is the best response to their opponent's best response—a response that provides the most favourable outcome to each respective player given the other player's strategies. As an example, in the game of Chicken, there is a Nash equilibrium at (Swerve, Straight). This is shown in Figure 4a. Player 2 has no incentive to switch their strategy from Straight to Swerve if Player 1 leaves their strategy unchanged. If Player 2 decided to swerve while Player 1 chose to swerve, Player 2 would stand to lose one util or dollar (depending on the interpretation of the payoff units). The experiment results are interpreted economically using these game theory concepts. The results section also comments on the role of Nash equilibria in the computational method applied in this model.

Basic Game Descriptions

In the Prisoner's Dilemma, two criminals are arrested and brought in to a police station. The police admit that they cannot charge either of them on the principal charge, but can charge them for a lesser crime with a sentence of one year. They offer the following bargain to each of the prisoners: if he testifies (defects) against (and incriminates) his partner, he will go free while his partner will get five years for the principal charge. However, if both testify against and incriminate each other, both prisoners will get two years in jail. The mutually most beneficial action is dominated. In my formulation of the Prisoner's Dilemma, the payoff of (Defect, Defect) is less than the payoff of (Cooperate, Cooperate); we can interpret this as a prisoner's utility derived from a lower jail sentence.

The Deadlock game is numerically equivalent to the Prisoner's Dilemma except that the mutually most beneficial action is dominant—it is optimal and in the best interest for each of the players to defect. This type of situation occurs when the two parties do not have any interest in cooperating with one another, but would rather have the other compromise; it may be common to diplomacy and international relations.

The game of Chicken involves two individuals driving along a single-lane stretch that approaches a bridge; each player can swerve to avoid the other, letting them take the bridge alone, or keep driving straight, risking a fatal head-on collision. The best strategy for each individual is to keep driving straight, while the other person swerves. Therefore, the mutually beneficial action is to play different strategies, which results in more than one Nash equilibrium.

The Stag Hunt differs from the aforementioned games because it contains two pure strategy Nash equilibria with symmetric payoffs, one that is risk dominant (when both players defect) and one that is payoff dominant (when both players cooperate). Formally, the game involves two hunters. Each hunter can individually hunt a stag or a hare. If one hunts a stag, he will require the other individual's cooperation to succeed. However, if one hunts a hare, he can succeed on his own, but a hare is worth less than a stag. Each player must act without knowing the choice of the other, making this a simultaneous coordination game.

The Battle of the Sexes game involves a couple that was supposed to meet for a night out at either the opera or a football game; neither can remember which event was the agreed upon meeting place. The husband would prefer to go to the football game, while the wife would prefer to go to the opera. However, both prefer to be in attendance together rather than alone. There are two versions of this game—Version 2 introduces a "cost" (disutility) that accounts for the fact that the husband and wife could each choose the event that is not their most preferred, while Version 1 does not. The former is called "Battle of the Sexes 2". The two pure strategy Nash equilibria occur when both go to the opera and both go to the football game.

Matching Pennies is the two-strategy version of Rock Paper Scissors. It is also mathematically equivalent to Odds and Evens. In this game, two individuals each have a penny.

Each must secretly turn the penny to heads or tails. If the faces of both pennies match, Player 1 keeps both pennies (and gains +1); if the faces do not match, Player 2 keeps both pennies (and gains +1). There is no pure strategy Nash equilibrium in this game. Instead, the Nash equilibrium is in mixed strategies when both players choose heads or tails with equal probabilities.

In Choosing Sides, two individuals are driving along a dirt road. One driver must swerve to avoid the other to avoid a head-on collision. It does not matter which side they choose, as long as it is the same side (both left or both right). The story that goes with Choosing Sides seems mathematically equivalent to the Battle of the Sexes. However, when the two individuals coordinate in Choosing Sides, they face an equal payoff structure. In the Battle of the Sexes, the two individuals prefer a certain activity to the other. Choosing Sides is also different from the Stag Hunt because it does not contain an activity that is "safer" than the other (i.e. one risks less when hunting a hare instead of a stag). As a result, Choosing Sides contains two pure strategy Nash equilibria that are Pareto efficient with equal payoffs.

The pure coordination game also differs from Choosing Sides because it contains only one pure strategy Nash equilibrium. The two individuals in this game prefer the same Nash equilibrium outcome, in which the payoff is the most mutually beneficial. In its classical formulation, the two players can either go to a party or stay home. Both prefer going to the party over staying at home, and both prefer to stay at home than engage in different activities. The (Party, Party) outcome dominates the (Home, Home) outcome as the (Home, Home) outcome dominates the non-coordinating outcomes (Party, Home) and (Home, Party).

A summary of the simultaneous game characteristics is provided in Figure 1 below. Games with two pure strategy Nash equilibria include Chicken, the Stag Hunt, Battle of the Sexes, and Choosing Sides; games with only one pure strategy Nash equilibrium include the Prisoner's Dilemma, Deadlock, and the pure coordination game. Matching Pennies and Rock Paper Scissors do not have any pure strategy Nash equilibria; due to their zero-sum nature, these games have unique Nash equilibria in mixed strategies. It should be noted that many of the games below are synonymous to other games, with identical outcomes but different names: the Diner's Dilemma is the Prisoner's Dilemma; the Volunteer's Dilemma is Chicken or Hawk-Dove; and Matching Pennies is like a two-strategy formulation of Rock Paper Scissors. This result allows game theorists to apply one simple model to an array of realistic examples of "player" interaction.

Figure 1: Summary of Game Characteristics

Game	Strategies Per Player	Number of Pure Strategy Nash Equilibria	Pure Strategy Nash Equilibria	Sequential	Perfect Information	Zero Sum
Prisoner's Dilemma	2	1	(D, D) = (1, 1)	No	No	No
Deadlock	2	1	(D, D) = (2, 2)	No	No	No
Chicken	2	2	(D, C) = (6, 2) (C, D) = (2, 6)	No	No	No
Stag Hunt	2	2	(C, C) = (2, 2) (D, D) = (1, 1)	No	No	No
Battle of the Sexes 1 & 2	2	2	(O, O) = (3, 2) (F, F) = (2, 3)	No	No	No
Matching Pennies	2	0	N/A	No	No	Yes
Rock, Paper, Scissors	3	0	N/A	No	No	Yes
Choosing Sides	2	2	(L, L) = (1, 1) (R, R) = (1, 1)	No	No	No
Pure Coordination	2	1	(P, P) = (2, 2)	No	No	No

Computational Method

Chapter 11 of Kendrick (2006) applies genetic algorithms to the study of evolutionary game theory. The textbook uses the Prisoner's Dilemma as an example game to simulate one-shot (repeated) games between two individuals; the game is repeated 24 times and the population consists of eight individuals, so each player plays the other seven players 24 times. For the purpose of easy convergence to equilibrium, I have increased the population size to 16 in my experiments. Additionally, I have modified the game loops in the MATLAB code to reflect the different payoff structures in each game. Genetic algorithms are search procedures and/or heuristics that mimic the process of natural selection and genetics; the rules of Darwin's "survival of the fittest" apply to this model in which individuals compete with one another and the fittest individuals form a couple in order to give birth to the next generation.

The genetic algorithm presented in these models is quite realistic in that it makes use of inheritance, mutation, crossover, and selection. Inheritance is evident in this model when the chromosome, containing a string of genes, for each individual in a population may contain similar genes to individuals in the populations from previous generations/runs. The mutation and crossover operations also provide a simple representation of those that can be found in the human race. Selection in the genetic algorithm is achieved by selecting the "best fit" agents in the population—here; the "best fit" strategies are the ones with the highest payoffs. In order for the genetic algorithm to begin, the genetic representation (the number of genes in each chromosome) and fitness function need to be defined. After this, the genetic algorithm can proceed to the following steps before finding a global optimum: initialization (generation of the initial population given some deterministic or stochastic criteria), selection (again, given deterministic or stochastic criteria), genetic operators (rate of mutation and the crossover method), and termination (when the algorithm finds the local or global optimum).

Experiments

My formulation of the different simultaneous games acts as more of a calibration exercise than an experiment; however, I think that this is still a useful extension to the economic and computational model because it allows me to compare the convergence to the equilibrium outcomes (if any) in each game. There are two main experiments carried out using the genetic algorithm. The first extends the number of runs to 500 for games that fail to converge in 100 runs. The second uses a deterministic method to initialize the population (as opposed to a random one) in order to start with a population consisting entirely of co-operators (a binary string of 24 ones); this requires changing the gene pool equation from genepool (k1) = ceil (rand*(2^clen)-1) to genepool (k1) = (2^clen)-1 in the initpoprand_gagagme.m file.

The first experiment is carried out on Matching Pennies and Rock, Paper, Scissors and the second experiment is conducted for the Prisoner's Dilemma, Deadlock, Chicken, and the Stag Hunt. Payoff matrices, graphs representing the fittest strategies, and a preliminary description of the computational results are provided in the results section; a more detailed analysis relating the economic theory to the model is provided in the Discussion section. Generally, the strategy in the first column (Cooperate) of the payoff matrix corresponds to a one in the chromosome string, while the strategy in the second column (Defect) corresponds to a zero in the chromosome string. In each game and experiment, the population size, chromosome length, and probability of mutation are held constant at 16, 24, and 0.7, respectively.

Results

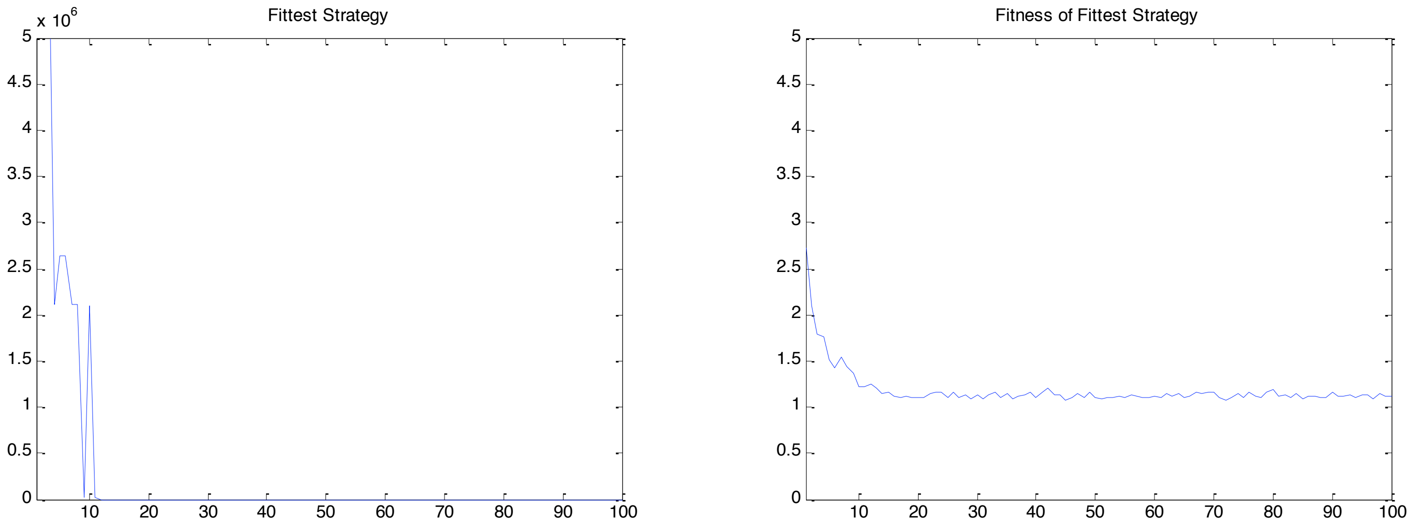
Prisoner's Dilemma

Figure 2a

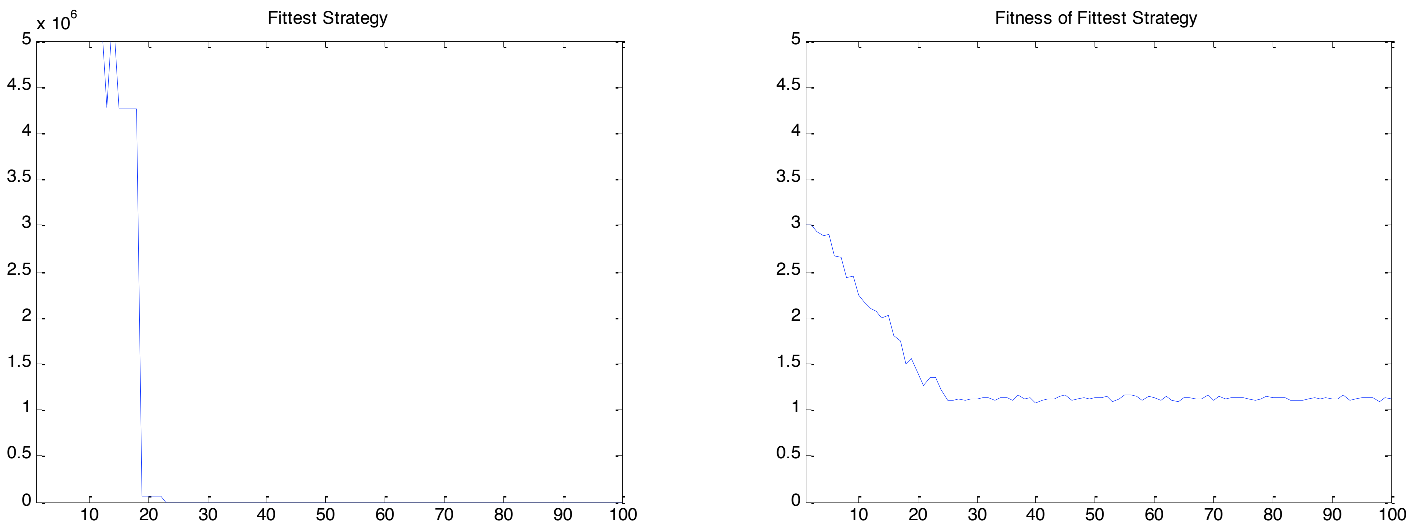
		Player 2	
		C	D
Player 1	C	3, 3	0, 5
	D	5, 0	**1, 1**

Figure 2b

Random Initial Population



Deterministic Initial Population



As can be seen from Figures 2a and 2b, the prisoner's dilemma formulation of the game converges to equilibrium around 15 runs/generations. The fittest strategy converges to zero, while the fitness of the fittest strategy converges to a payoff of one, indicating that the best strategy is for all players to defect (evidently, the Nash equilibrium). The results also show the fittest strategy and the fitness of that strategy when the initial population consists entirely of cooperators. It is interesting that even when the population consists entirely of cooperators, the fittest strategy still converges to defection, albeit more slowly (at around 20 runs) than under the randomized initial population.

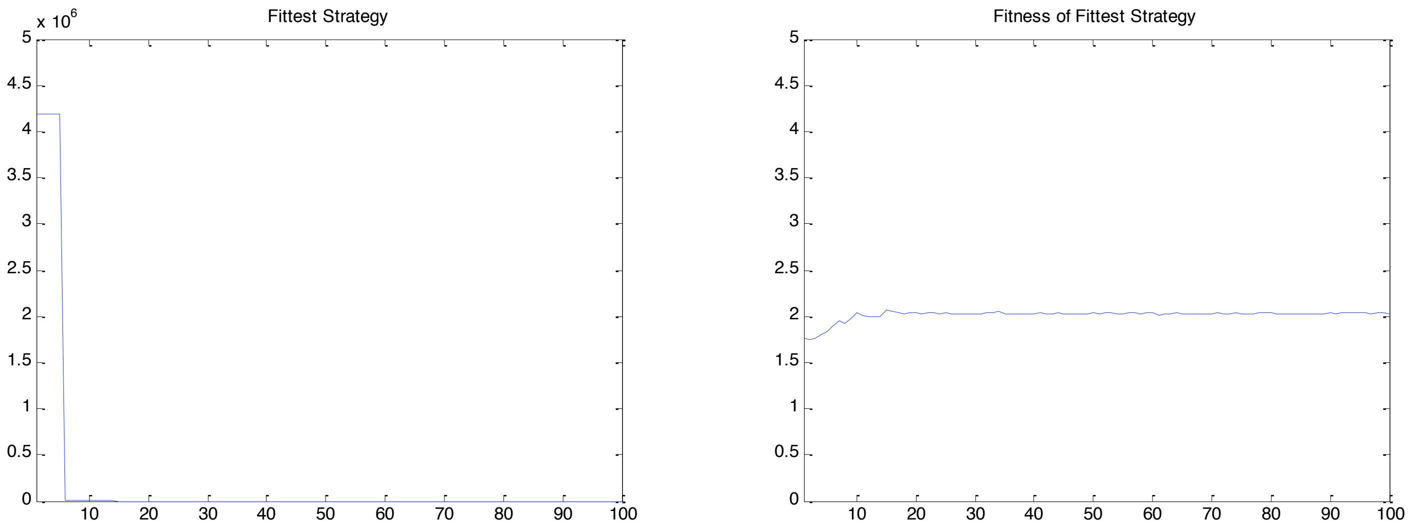
Deadlock

Figure 3a

		Player 2	
		C	D
Player 1	C	1, 1	0, 3
	D	3, 0	**2, 2**

Figure 3b

Random Initial Population



Deterministic Initial Population

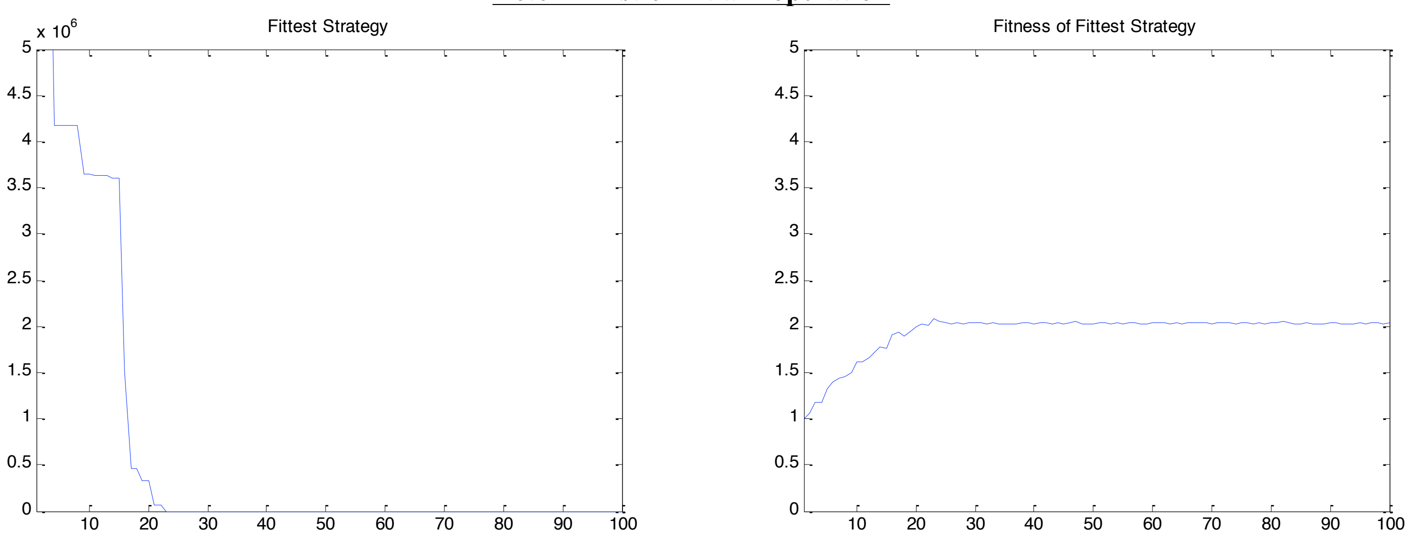


Figure 3b shows the convergence to the fittest strategy for the Deadlock game. Unlike in the Prisoner's Dilemma, the strategy converges to defection almost immediately, after about five runs. It seems that because the Nash equilibrium strategy is also the most mutually beneficial outcome, subsequent generations are able to adapt their strategies relatively quickly. When the initial population is generated deterministically instead of randomly, consisting entirely of cooperators, the fittest strategy reaches convergence after a larger number of runs, as is evident in the Prisoner's Dilemma.

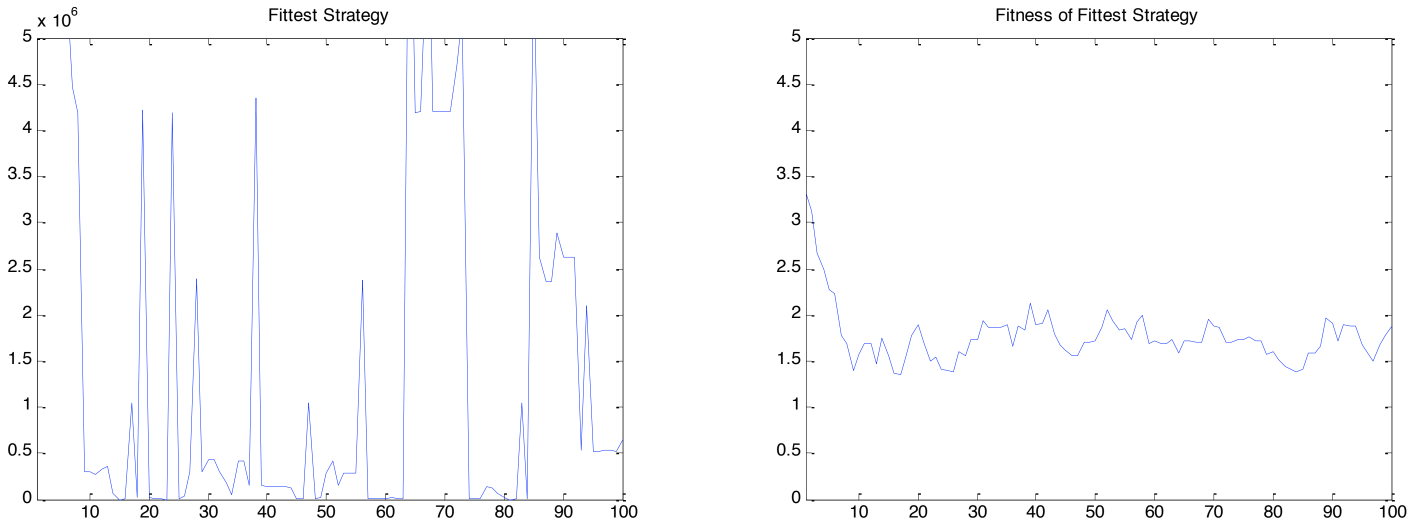
Chicken

Figure 4a

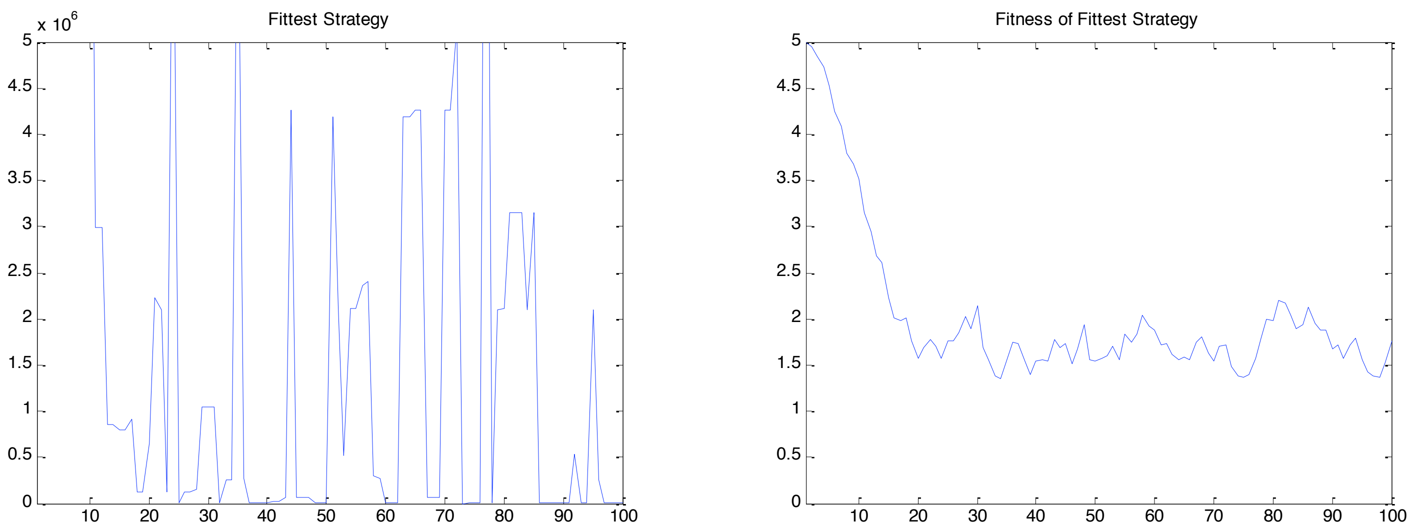
		Player 2	
		Swerve	Straight
Player 1	Swerve	5, 5	**2, 6**
	Straight	**6, 2**	1, 1

Figure 4b

Random Initial Population



Deterministic Initial Population



The fitness of the fittest strategy under the chicken game does not converge to any stable equilibrium. There are persistent mutations at all generations of the model under the randomly and deterministically initialized populations. However, the strategy does tend to bounce around the 1.5 to 2.0 neighbourhood (as can be seen in the top and lower right panels) which is somewhat consistent with the presence of two Nash equilibria in the chicken model—one at

(Swerve, Straight) and one at (Straight, Swerve). It seems that a mixed strategy for each of the players is the best response.

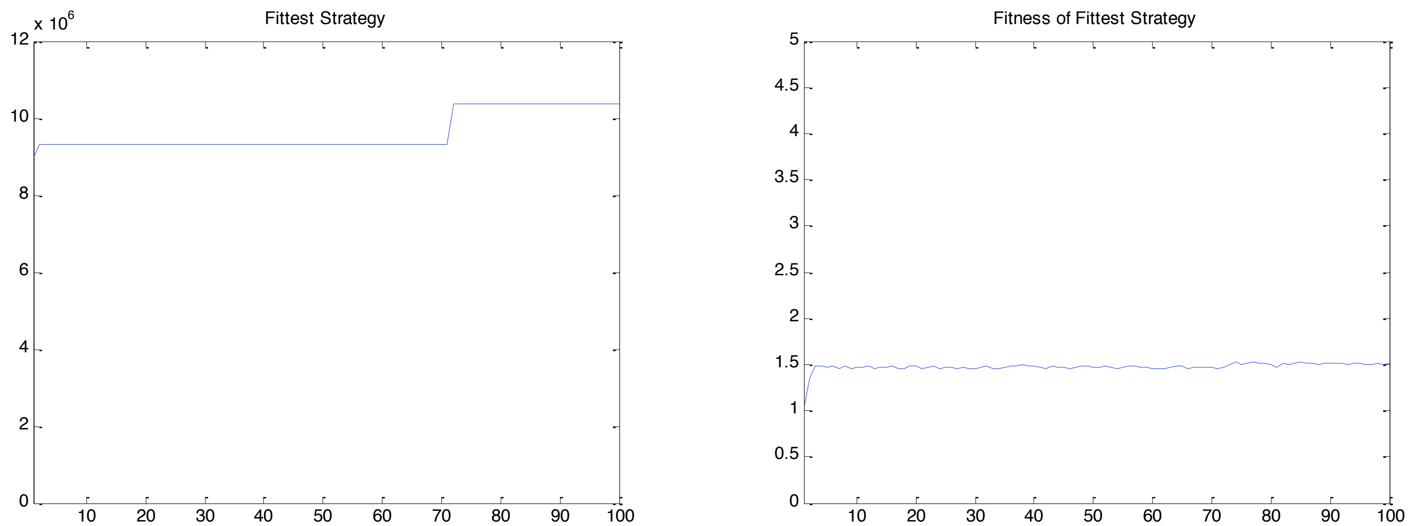
Stag Hunt

Figure 5a

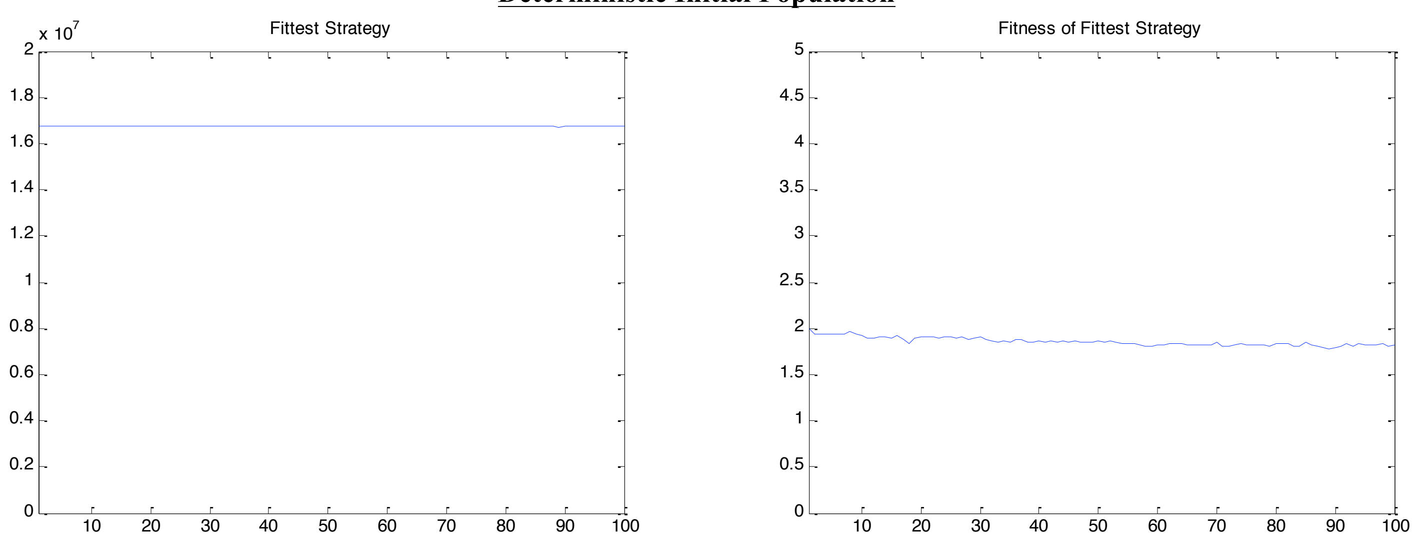
		Player 2	
		Stag	Hare
Player 1	Stag	**2, 2**	0, 1
	Hare	1, 0	**1, 1**

Figure 5b

Random Initial Population



Deterministic Initial Population



The convergence to the fittest strategy in the Stag Hunt game is relatively more stable; there is only one significant mutation present at 70 runs. Under the randomized initial population, the fitness of the fittest strategy is 1.5, indicating that the payoff to each individual is equal to the average payoff between the two Nash equilibria [i.e. E (payoff) = 0.5(2) + 0.5(1) = 1.5]. In the deterministic initial population simulation, the fittest strategy is for both parties to cooperate. This result is not surprising as it reflects the theoretical basis of the Stag Hunt: if the players start the game cooperating, they have no incentive to defect because they will only receive a payoff of either zero or one.

Battle of the Sexes

Figure 6a

Battle of the Sexes 1

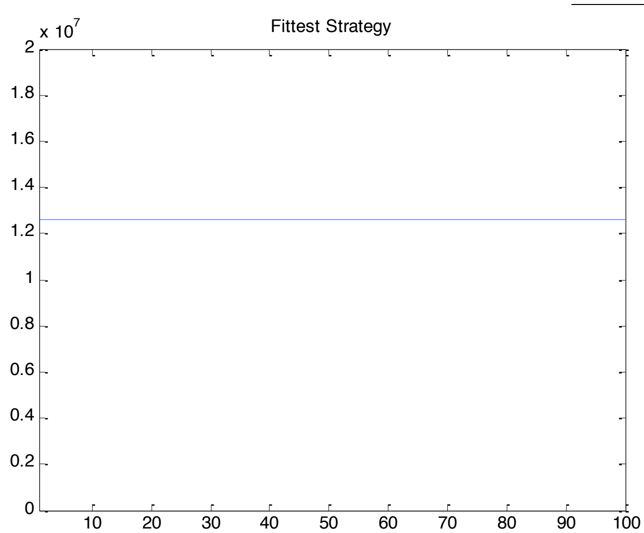
		Husband	
		Opera	Football
Wife	Opera	**3, 2**	0, 0
	Football	0, 0	**2, 3**

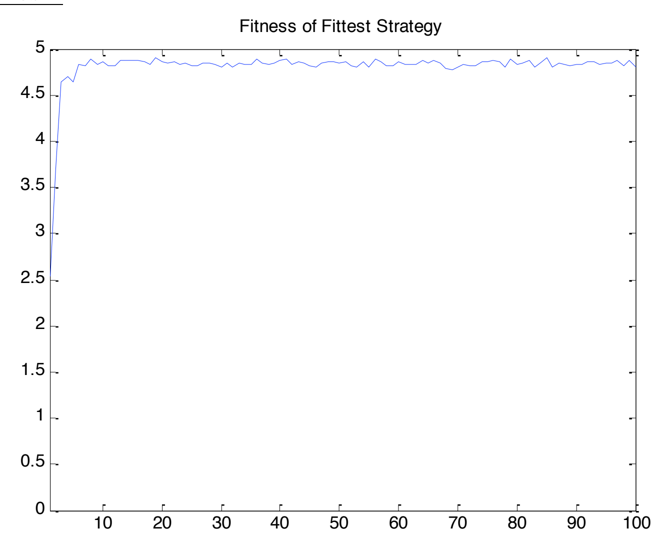
Battle of the Sexes 2

		Husband	
		Opera	Football
Wife	Opera	**3, 2**	1, 1
	Football	0, 0	**2, 3**

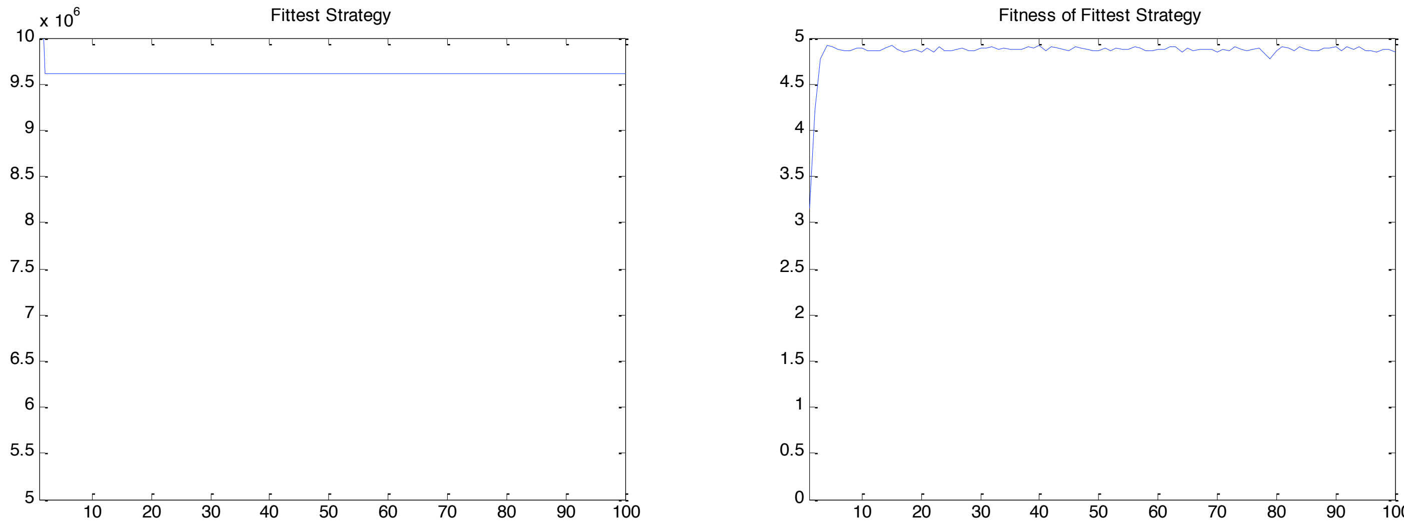
Figure 6b

Battle of the Sexes 1





Battle of the Sexes 2



The results from both formulations of the Battle of the Sexes are inherently similar. The only difference between these two games is the cost associated with the husband and wife choosing a meeting location that is not their first preference. Because of this "added cost" in the Battle of the Sexes 1, with a payoff structure of (0, 0) at (Opera, Football), the fitness of the fittest strategy converges more slowly than that of the Battle of the Sexes 2. The fitness converges at around eight runs in version 1 and around four runs in version 2 of this game.

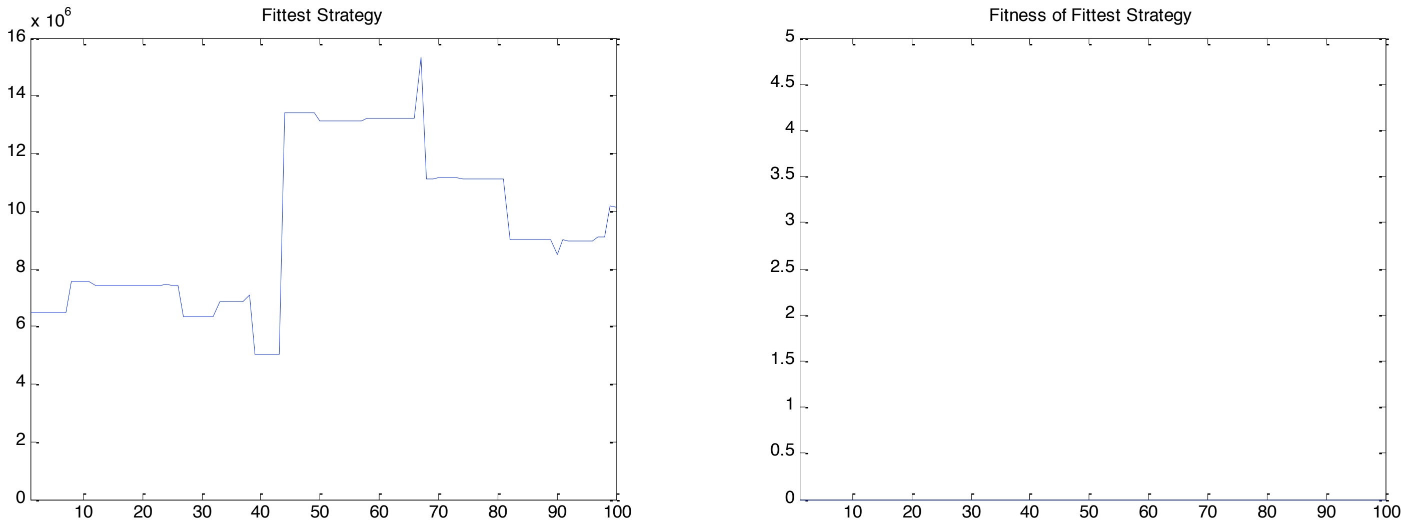
Matching Pennies

Figure 7a

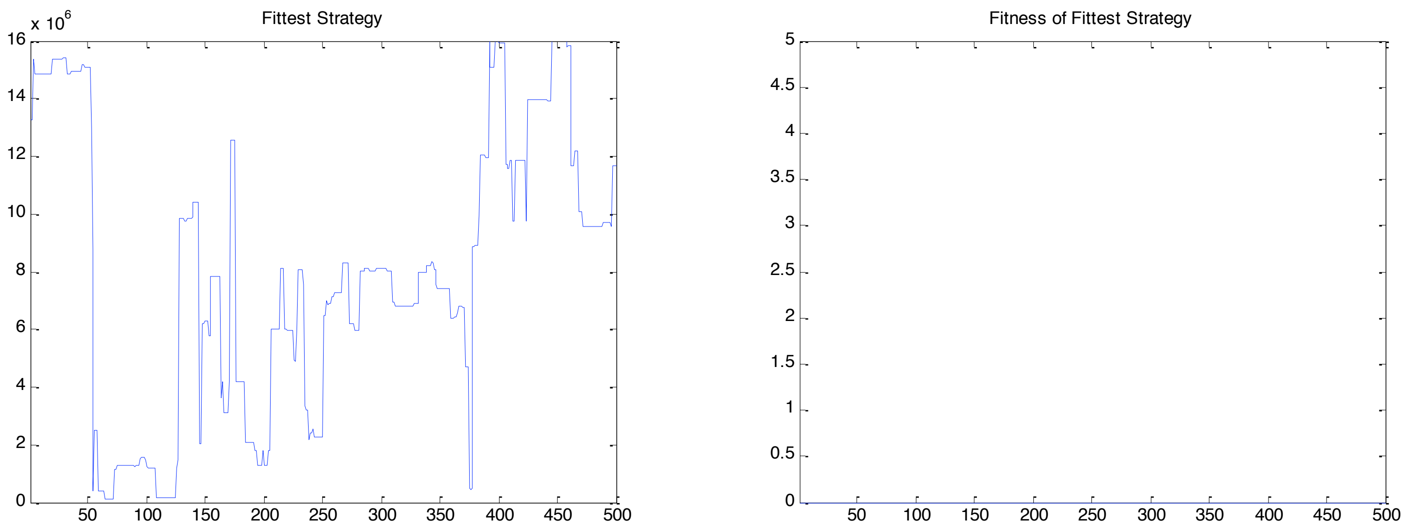
		Player 2	
		Heads	Tails
Player 1	Heads	1, -1	-1, 1
	Tails	-1, 1	1, -1

Figure 7b

Number of Runs = 100



Number of Runs = 500



The results from the Matching Pennies game are somewhat more interesting. Because this game is absent of any pure strategy Nash equilibria, a fittest strategy is never obtained, even after 500 runs. It is also interesting to note that, because this game is also a zero-sum game, the fitness of the fittest strategy immediately converges to zero in both the 100 and 500 run simulations. This result is consistent with each player opting for a mixed strategy approach, choosing Heads or Tails with equal probability.

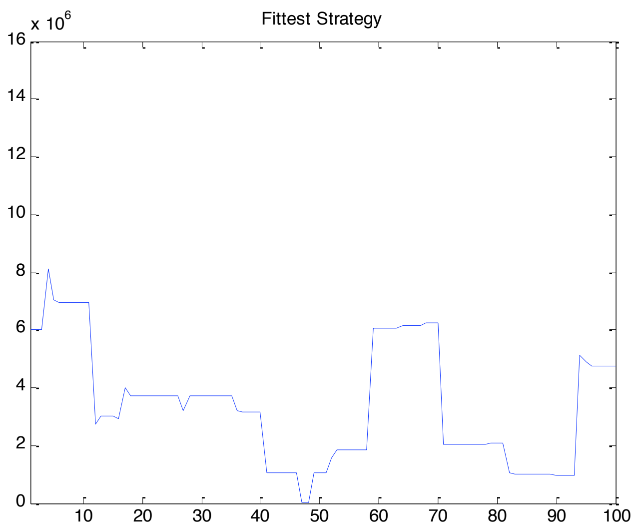
Rock, Paper, Scissors

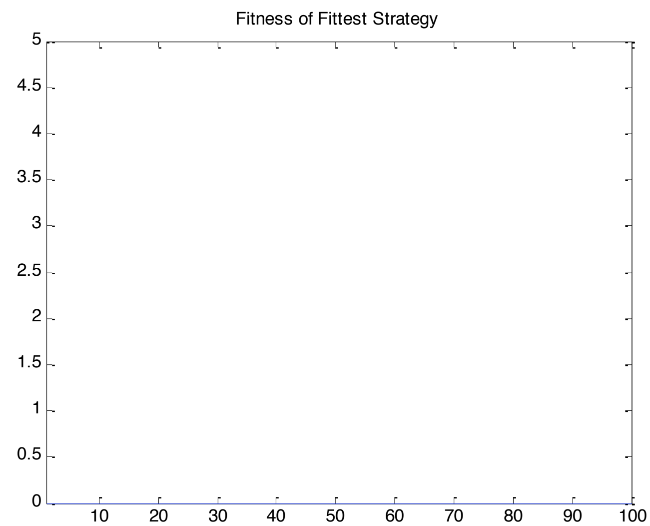
Figure 8a

		Player 2		
		Rock	Paper	Scissors
Player 1	Rock	0, 0	-1, 1	1, -1
	Paper	1, -1	0, 0	-1, 1
	Scissors	-1, 1	1, -1	0, 0

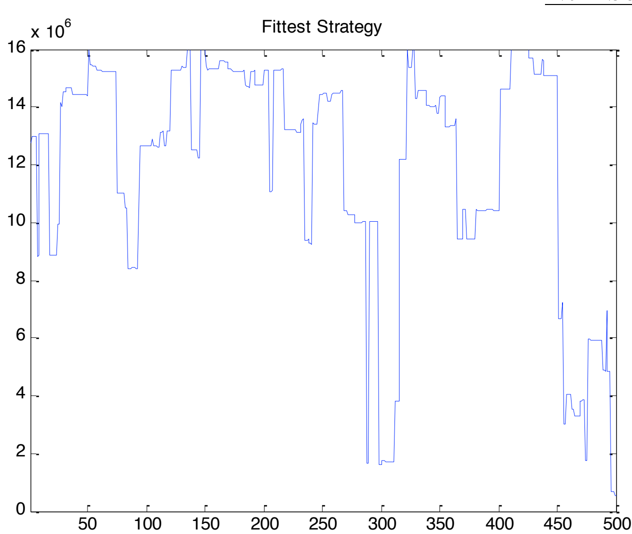
Figure 8b

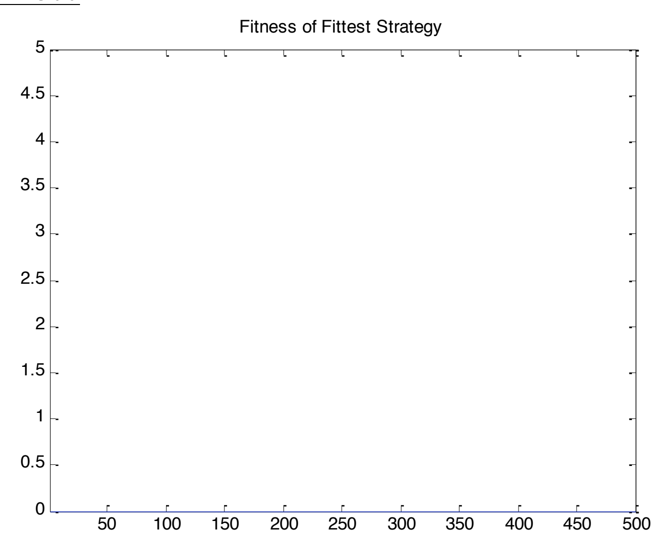
Number of Runs = 100





Number of Runs = 500





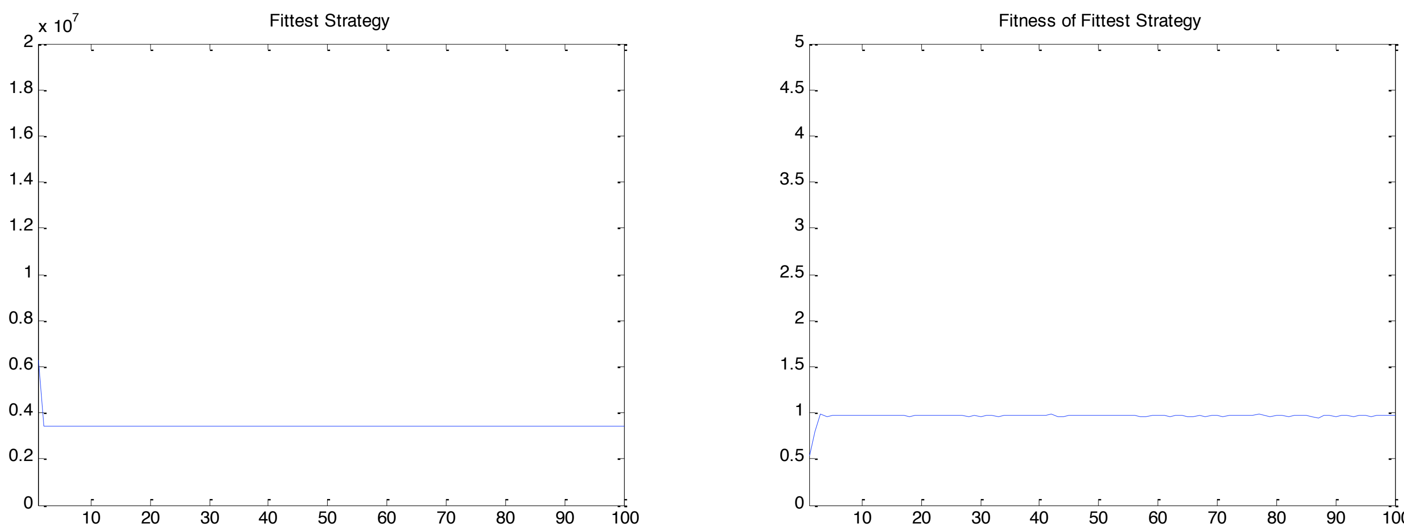
The results from Rock, Paper, Scissors are almost identical to those from Matching Pennies. This is because Rock, Paper, Scissors is the three-strategy version of Matching Pennies, thus, containing the same game characteristics—a zero-sum game with no pure strategy Nash equilibria and players employing a mixed strategy approach.

Choosing Sides

Figure 9a

		Player 2	
		Left	Right
Player 1	Left	**<u>1, 1</u>**	0, 0
	Right	0, 0	**<u>1, 1</u>**

Figure 9b



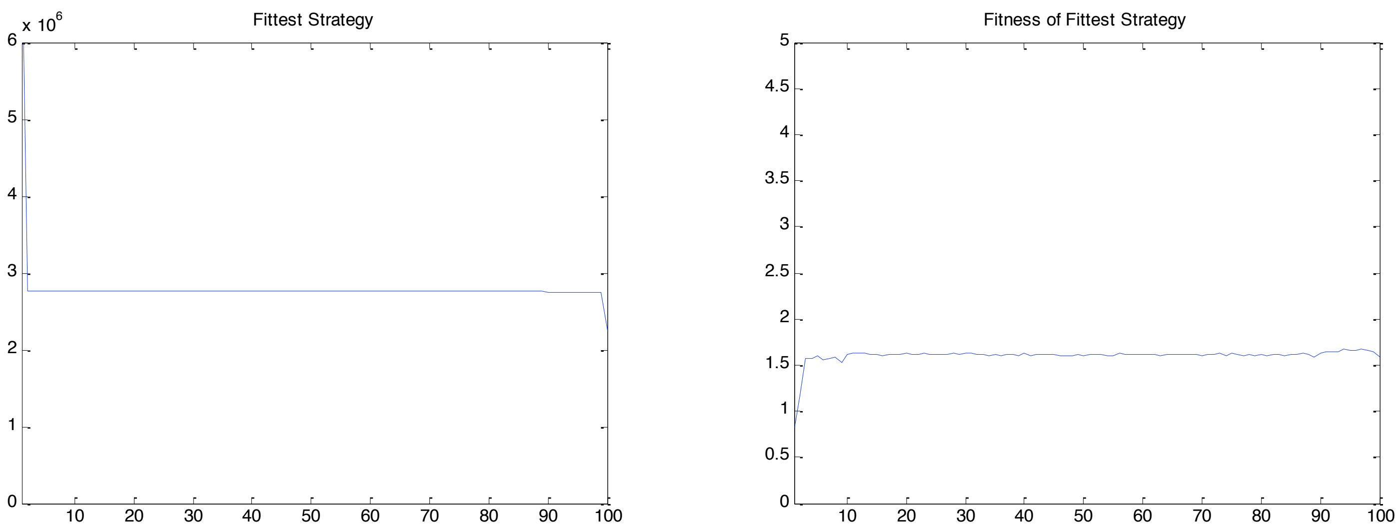
The results for the Choosing Sides game are somewhat trivial. As mentioned in the game descriptions earlier, this game only requires the two players to choose the same action—both Left or both Right—which results in two pure Nash equilibria. The decimal representation of the chromosome of the fittest individual is shown in the first graph; the results indicate that the fittest individual employs a mixed strategy, as the chromosome representation does not converge to zero. In the second graph, we see that the fitness of this strategy converges to one, which reflects the payoffs at the two equilibria.

Pure Coordination

Figure 10a

		Player 2	
		Party	Home
Player 1	Party	**2, 2**	0, 0
	Home	0, 0	**1, 1**

Figure 10b



The final simulation in this analysis models a pure coordination game. Two individuals need to decide if they should go to a party or stay home. This game contains two Nash equilibria: one at (Party, Party) and one at (Home, Home). Since each individual would rather go to the party than stay home, the (Party, Party) outcome dominates the (Home, Home) outcome. The presence of these two Nash equilibria can be seen in the second graph. The fitness of the fittest strategy converges to 1.5, which represents the individual's average payoff in the presence of two Nash equilibria—that is, E (payoff) = 0.5(2) + 0.5(1) = 1.5.

Discussion

Results and Economic Model/Computational Method

As is evident in the results presented above, modeling different games with a varying number of Nash equilibria and corresponding "payoffs" or agent utility is particularly useful in analyzing the path and time of strategy convergence. We can see that if a game contains two pure strategy Nash equilibria, the path to convergence, as portrayed in the "Fittest Strategy" graphs, is significantly more volatile than in games with only one Nash equilibrium. We can also see that the fitness of this fittest strategy (the corresponding payoffs) tends to bounce between a specific range, indicating a mixed strategy approach from the players. This type of model behaviour is simulated in Chicken, the Stag Hunt, the Battle of the Sexes, Choosing Sides and the pure coordination game.

When the game contains only one Nash equilibrium, as is the case of the Prisoner's Dilemma and Deadlock, the path to convergence in the strategy (defection) is more immediate. Additionally, the fitness of this strategy quickly reduces to the corresponding matrix payoffs; for example, the fitness of the fittest strategy quickly reduces to one in the Prisoner's Dilemma and two in Deadlock. When the game does not contain a Nash equilibrium, we can see that a "fittest strategy" is not identified—the graph of the fittest strategy has several large deviations. This type of game is modeled in Matching Pennies, or equivalently, in Rock, Paper, Scissors.

It seems that the formulation of the genetic algorithm has a strong influence on these results. With each successive run, the "best fit" individual and corresponding chromosome string survives simply because of its arbitrary fitness level (determined initially by the randomization of the population and the binary representation). Since a player's actions are determined regardless of their opponent's actions, the model simulates a situation in which it is in the best interest of each individual agent to maximize their own payoff and ignore the most mutually beneficial outcome. The introduction of chromosome crossover and mutation seem to have little influence on maintaining a population of co-operators, unless the initial population consists entirely of co-operators.

Advantages and Limitations (of genetic algorithms in general)

Advantages

Genetic algorithms can solve a wide range of optimization problems as long as the problem can be described over the "chromosome" encoding. Depending on the deterministic or random aspects of the model, there may be several solutions to one problem, which provides practitioners with many alternatives (but which could be seen as a disadvantage to some researchers). There are many other advantages of genetic algorithms that allow for ease of implementation: the problems applied to GAs can be multidimensional, non-differential, non-continuous or non-parametric; bad proposals/solutions in the population are not an issue since the algorithm is capable of discarding them; and the algorithm itself does not need to know the rules/constraints of the problem, a feature which is particularly useful for loosely defined or complex problems.

Limitations

One of the limitations of genetic algorithms is that they must use approximated fitness levels to allow for computational efficiency. However, it has been found that the amalgamation of several models with approximated fitness is still promising. Sometimes when the convex problem is broken up into small parts, and these parts have become evolved, it is difficult to prevent them from mutation especially if they are required to combine well with other parts. For example, when designing a floor plan or an engine, it may be feasible to use genetic algorithms to generate shapes or design fan blades, but not to design the entire structure of these items with one algorithm. Additionally, the termination criterion is not clear in every problem since the "best" solution only seems "best" when compared to the other surrounding solutions.

Specifically, in convex problems, the algorithm may get stuck in a local optimum instead of reaching a global one; it does not know how to sacrifice short-term fitness for long-term fitness.

Further Research

As the games presented in this analysis are only one-shot (simultaneous) games, it would be particularly interesting for game theory empiricists to model a sequential game, such as chess, backgammon, tic-tac-toe, or Go. It would also be possible to model a simple, two-player interaction game that consists of only two strategies: left and right. This type of game is provided as an example in the introductory game theory portion of ECON 452 (Information and Incentives). We used a decision tree with Player 1 as the "leader" and Player 2 as the "Follower" to find the subgame perfect Nash equilibrium and the non-credible threat. In reference to the game of Chicken, an example of a non-credible threat involves one of the drivers ripping the steering wheel from their car. This action does not result in a credible threat since the wheel-ripper's opponent can always swerve knowing that the other has removed their steering wheel and cannot possibly swerve. Although the player that has not yet removed their wheel has that option, it is not a credible threat since it would require harming themselves, preventing the combination of strategies (Straight, Straight).

Future researchers may wish to model the games presented here in a neural network representation. They may also wish to model "learning" in the agents' behaviour, as well as more complex social interactions between the players with family, friends, or neighbours. As mentioned in Kendrick (2006), including strategic thinking and behaviour—when a player's actions are reactions to the opponent's past actions—in the players would not result in a population of all defectors; the fittest strategy may converge to a population of all co-operators or show complex cyclical behaviour.

Appendix – MATLAB Code

Generation of Initial Population, Number of Runs, and Mutation Rate:

```
% FINAL PROJECT CALIBRATION AND EXPERIMENTS BY MONICA MOW
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Genetic Algorithm evolutionary game program
% with initpopdet and parentsdet
% Program name: gagame.m
% On an early version by Huber Salas
clear all;
% initialization of counters and parameters;
nruns = 100;   popsize = 16;

clen = 24;   pmut = 0.7;

% generation of chromosome strings of initial population
genepool= initpoprand_gagame(popsize,clen);
```

For 500 runs:

```
% Genetic Algorithm evolutionary game program
% with initpopdet and parentsdet
% Program name: gagame.m
% On an early version by Huber Salas
clear all;
% initialization of counters and parameters;
nruns = 500;   popsize = 16;

clen = 24;   pmut = 0.7;
```

The fitness files were modified for each game (PD is Prisoner's Dilemma):

```
for k = 1:nruns;

    % computation of fitness function and fittest individual
    [fit, bestind, bestfit] = fitness_gagamePD(genepool,popsize,clen);
    wbest(k) = bestind;
    fbest(k) = bestfit;
```

With the population of all co-operators, the initpoprand_gagame_COOP function is substituted into the original code:

```
function genepool= initpoprand_gagame(popsize,clen);
for k1 = 1:popsize;
genepool(k1) =(2^clen)-1;
%genepool(k1) =  (2^clen)-1;
dec2bin(genepool(k1), clen)
end
```

```
% generation of chromosome strings of initial population
genepool= initpoprand_gagame_COOP(popsize,clen);
```

Loops for the different games:

Prisoner's Dilemma

```
%Loop for games
for k3 = 1:clen;
actionp1 = bitand(strategyp1,mask);
actionp2 = bitand(strategyp2,mask);
mask = bitshift(mask,1);
    % Prisoner's dilemma payoffs
    % defect, defect
    if (actionp1 == 0) & (actionp2 == 0)
        payoffs(k1) = payoffs(k1) + 1;
    end
    % cooperate, defect
    if (actionp1 > 0) & (actionp2 == 0)
        payoffs(k1) = payoffs(k1) + 0;
    end
    % defect, cooperate
    if (actionp1 == 0) & (actionp2 > 0)
        payoffs(k1) = payoffs(k1) + 5;
    end
    % cooperate, cooperate
    if (actionp1 > 0) & (actionp2 > 0)
        payoffs(k1) = payoffs(k1) + 3;
    end
end % end loop games
```

Deadlock

```
%Loop for games
for k3 = 1:clen;
actionp1 = bitand(strategyp1,mask);
actionp2 = bitand(strategyp2,mask);
mask = bitshift(mask,1);
    % Deadlock payoffs
    % defect, defect
    if (actionp1 == 0) & (actionp2 ==
        payoffs(k1) = payoffs(k1) + 2;
    end
    % cooperate, defect
    if (actionp1 > 0) & (actionp2 == (
        payoffs(k1) = payoffs(k1) + 0;
    end
    % defect, cooperate
    if (actionp1 == 0) & (actionp2 > (
        payoffs(k1) = payoffs(k1) + 3;
    end
    % cooperate, cooperate
    if (actionp1 > 0) & (actionp2 > 0)
        payoffs(k1) = payoffs(k1) + 1;
    end
end % end loop games
```

Chicken	Stag Hunt

Chicken

```
%Loop for games
for k3 = 1:clen;
actionp1 = bitand(strategyp1,mask);
actionp2 = bitand(strategyp2,mask);
mask = bitshift(mask,1);
    % Chicken payoffs
    % defect, defect
    if (actionp1 == 0) & (actionp2 == 0
        payoffs(k1) = payoffs(k1) + 1;
    end
    % cooperate, defect
    if (actionp1 > 0) & (actionp2 == 0)
        payoffs(k1) = payoffs(k1) + 2;
    end
    % defect, cooperate
    if (actionp1 == 0) & (actionp2 > 0)
        payoffs(k1) = payoffs(k1) + 6;
    end
    % cooperate, cooperate
    if (actionp1 > 0) & (actionp2 > 0)
        payoffs(k1) = payoffs(k1) + 5;
    end
end % end loop games
```

Stag Hunt

```
%Loop for games
for k3 = 1:clen;
actionp1 = bitand(strategyp1,mask);
actionp2 = bitand(strategyp2,mask);
mask = bitshift(mask,1);
    % Stag hunt payoffs
    % defect, defect
    if (actionp1 == 0) & (actionp2 == (
        payoffs(k1) = payoffs(k1) + 1;
    end
    % cooperate, defect
    if (actionp1 > 0) & (actionp2 == 0)
        payoffs(k1) = payoffs(k1) + 0;
    end
    % defect, cooperate
    if (actionp1 == 0) & (actionp2 > 0)
        payoffs(k1) = payoffs(k1) + 1;
    end
    % cooperate, cooperate
    if (actionp1 > 0) & (actionp2 > 0)
        payoffs(k1) = payoffs(k1) + 2;
    end
end % end loop games
```

Battle of the Sexes 1

```
%Loop for games
for k3 = 1:clen;
actionp1 = bitand(strategyp1,mask);
actionp2 = bitand(strategyp2,mask);
mask = bitshift(mask,1);
    % Battle of the sexes payoffs
    % Opera, Opera
    if (actionp1 == 0) & (actionp2 == 0)
        payoffs(k1) = payoffs(k1) + 3;
    end
    % Opera, Football
    if (actionp1 > 0) & (actionp2 == 0)
        payoffs(k1) = payoffs(k1) + 0;
    end
    % Football, Opera
    if (actionp1 == 0) & (actionp2 > 0)
        payoffs(k1) = payoffs(k1) + 0;
    end
    % Football, Football
    if (actionp1 > 0) & (actionp2 > 0)
        payoffs(k1) = payoffs(k1) + 2;
    end
    % Opera, Opera
    if (actionp1 == 0) & (actionp2 == 0)
        payoffs(k2) = payoffs(k2) + 2;
    end
    % Opera, Football
    if (actionp1 > 0) & (actionp2 == 0)
        payoffs(k2) = payoffs(k2) + 0;
    end
    % Football, Opera
    if (actionp1 == 0) & (actionp2 > 0)
        payoffs(k2) = payoffs(k2) + 0;
    end
    % Football, Football
    if (actionp1 > 0) & (actionp2 > 0)
        payoffs(k2) = payoffs(k2) + 3;
    end
end % end loop games
```

Battle of the Sexes 2

```
%Loop for games
for k3 = 1:clen;
actionp1 = bitand(strategyp1,mask);
actionp2 = bitand(strategyp2,mask);
mask = bitshift(mask,1);
    % Battle of the sexes 2 payoffs
    % Opera, Opera
    if (actionp1 == 0) & (actionp2 ==
        payoffs(k1) = payoffs(k1) + 3;
    end
    % Opera, Football
    if (actionp1 > 0) & (actionp2 == 0
        payoffs(k1) = payoffs(k1) + 1;
    end
    % Football, Opera
    if (actionp1 == 0) & (actionp2 > 0
        payoffs(k1) = payoffs(k1) + 0;
    end
    % Football, Football
    if (actionp1 > 0) & (actionp2 > 0)
        payoffs(k1) = payoffs(k1) + 2;
    end
    % Opera, Opera
    if (actionp1 == 0) & (actionp2 ==
        payoffs(k2) = payoffs(k2) + 2;
    end
    % Opera, Football
    if (actionp1 > 0) & (actionp2 == 0
        payoffs(k2) = payoffs(k2) + 1;
    end
    % Football, Opera
    if (actionp1 == 0) & (actionp2 > 0
        payoffs(k2) = payoffs(k2) + 0;
    end
    % Football, Football
    if (actionp1 > 0) & (actionp2 > 0)
        payoffs(k2) = payoffs(k2) + 3;
    end
end % end loop games
```

Matching Pennies* (100, 500)

```
%Loop for games
for k3 = 1:clen;
actionp1 = bitand(strategyp1,mask);
actionp2 = bitand(strategyp2,mask);
mask = bitshift(mask,1);
    % Matching pennies payoffs
    % defect, defect
    if (actionp1 == 0) & (actionp2 == 0)
        payoffs(k1) = payoffs(k1) + 1;
    end
    % cooperate, defect
    if (actionp1 > 0) & (actionp2 == 0)
        payoffs(k1) = payoffs(k1) - 1;
    end
    % defect, cooperate
    if (actionp1 == 0) & (actionp2 > 0)
        payoffs(k1) = payoffs(k1) - 1;
    end
    % cooperate, cooperate
    if (actionp1 > 0) & (actionp2 > 0)
        payoffs(k1) = payoffs(k1) + 1;
    end
    % defect, defect
    if (actionp1 == 0) & (actionp2 == 0)
        payoffs(k2) = payoffs(k2) - 1;
    end
    % cooperate, defect
    if (actionp1 > 0) & (actionp2 == 0)
        payoffs(k2) = payoffs(k2) + 1;
    end
    % defect, cooperate
    if (actionp1 == 0) & (actionp2 > 0)
        payoffs(k2) = payoffs(k2) + 1;
    end
    % cooperate, cooperate
    if (actionp1 > 0) & (actionp2 > 0)
        payoffs(k2) = payoffs(k2) - 1;
    end
end % end loop games
```

***This code is modified for the Rock, Paper, Scissors game. They are essentially identical.**

Choosing Sides

```
%Loop for games
for k3 = 1:clen;
actionp1 = bitand(strategyp1,mask);
actionp2 = bitand(strategyp2,mask);
mask = bitshift(mask,1);
    % Choosing sides payoffs
    % left, left
    if (actionp1 == 0) & (actionp2 == (
        payoffs(k1) = payoffs(k1) + 1;
    end
    % left, right
    if (actionp1 > 0) & (actionp2 == 0)
        payoffs(k1) = payoffs(k1) + 0;
    end
    % right, left
    if (actionp1 == 0) & (actionp2 > 0)
        payoffs(k1) = payoffs(k1) + 0;
    end
    % right, right
    if (actionp1 > 0) & (actionp2 > 0)
        payoffs(k1) = payoffs(k1) + 1;
    end
end % end loop games
```

Pure Coordination

```
%Loop for games
for k3 = 1:clen;
actionp1 = bitand(strategyp1,mask);
actionp2 = bitand(strategyp2,mask);
mask = bitshift(mask,1);
    % Pure coordination payoffs
    % party, party
    if (actionp1 == 0) & (actionp2 == 0
        payoffs(k1) = payoffs(k1) + 2;
    end
    % party, home
    if (actionp1 > 0) & (actionp2 == 0)
        payoffs(k1) = payoffs(k1) + 0;
    end
    % home, party
    if (actionp1 == 0) & (actionp2 > 0)
        payoffs(k1) = payoffs(k1) + 0;
    end
    % home, home
    if (actionp1 > 0) & (actionp2 > 0)
        payoffs(k1) = payoffs(k1) + 1;
    end
end % end loop games
```